tageous. The situation is analogous to that which exists for forces derivable from a potential. It is not necessary to introduce the potential function, but it can certainly be helpful at times.

The preceding statements are not confined to the simple system we have been considering but can easily be generalized to more complicated systems. Kane has himself provided us with an example that illustrates this. In Ref. 1 he applies both his method and the Gibbs-Appell method to a very complex problem, and on the basis of his results concludes that the labor involved in applying the Gibbs-Appell equation greatly exceeds the labor involved in applying his equations. However, in his calculations Kane failed to take advantage of the option described in the preceding paragraph. In the sentence immediately following Eq. (93) in Ref. 1, where, in the application of the Gibbs-Appell equations, he is faced with the necessity of taking the derivative of the square of a very complicated term, he assumes that it is necessary first to complete the square and then to take the derivative. Had he at this point implicitly differentiated, he would have ended up having to carry out exactly the same operations as required in the application of his own equations and would consequently have demonstrated that the labor involved starting with the Gibbs-Appell equations was the same as the labor involved starting with his equations. Since it was the results in Ref. 1 that ultimately led Kane to use the term "Kane's equations,"[2] one wonders what would have happened had he not, on the basis of this unfortunate oversight, erroneously concluded that the Gibbs-Appell method was more laborious than his own method.

It should be noted in our derivation of the Gibbs-Appell equation that none of the arguments depended on using the concept of virtual displacements or virtual work. Although it is not necessary to use these concepts in deriving or applying either Kane's equation or the Gibbs-Appell equation, they are nevertheless at times very helpful.

IV. Conclusion

From the preceding arguments, we conclude that Kane's equations are simply a particular form of the Gibbs-Appell equations and that Kane's method is simply a particular method of applying the Gibbs-Appell equations.

Although Kane cannot be credited with creating Kane's equations, he is responsible for resurrecting them, exploiting them, and promoting their use.

Concerning terminology, we suggest that Eq. (6) be called either Kane's form of the Gibbs-Appell equation or the Gibbs-Appell-Kane equation. We also suggest that Eq. (9) be called the Gibbs-Appell equation, which is the term, following Pars,[12] we have adopted. Most authors refer to Eq. (9) as Appell's equation. Kane, however, contends that Eq. (9) is "erroneously attributed to Appell"[1] and calls it Gibbs' equation.

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Rotation of a Triaxial Satellite near the Lagrangian Point L_4

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Introduction

IN recent years, the problem of the motion of a rigid body placed in the libration points of the restricted three-body problem has attracted the attention of several authors in relation to the possible use of such points in space flights. In those papers, one of the basic assumptions is that the center of the masses of the satellite remains exactly at the libration points.

This problem was started by Kane and Marsh,[1] who considered an axisymmetric satellite whose axis of symmetry is perpendicular to the orbital plane and made a study of the stability of the motion. This work was continued by Robinson,[2-4] who obtained the attitude stability for a triaxial satellite. Barkin[5] obtained stationary solutions to the rotation of the satellite placed at the Lagrangian point L_4 of the Earth-moon system by integrating the Eulerian equations of motion. Also, he found some families of periodic orbits, together with the conditions for their stability. Barkin and Markov[6] presented the equations of motion in terms of Delaunay-Hill's averaging scheme for the resonant case. An analogous study was made by Sidlichovsky,[7] who presented the solution of the first-order equations of the Lie-Hori theory. Gamarnik[8] and Krasilnikov[9] made some studies supporting the thesis that the center of masses moves along an arbitrary periodic orbit near the point L_4. Markeev[10] made an analogous study for the collinear point L_2. Elipe and Ferrer[11] analyzed the nonresonant case of an axisymmetric rigid body by means of the Lie-Deprit perturbation method, when the center of mass O_3 is moving in a neighborhood of L_4. In the resonant case, it is shown that the Hamiltonian is reduced to a generalized ideal resonance problem.

In the present Note, we give a first-order solution for the triaxial satellite, applying the Cid et al.[12] solution for the orbital motion. The solution for the axisymmetric rigid body mentioned above is obtained here as a particular case.

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Torque-Free Motion and Election of Variables

Let us consider the restricted circular three-body problem, in which the primaries O_1, O_2 of masses m_1, m_2 rotate around their mass center O in circular orbits and the satellite, which does not perturb the motion of the primaries, is a rigid body so that its center of masses is placed in a neighborhood of the Lagrangian point L_4.

The kinetic energy of the satellite in the synodic system can be expressed in terms of Andoyer's canonical variables (l,g,h,L,G,H), given in Ref. 13 as

$$F_0 = \tfrac{1}{2}\left(\frac{\sin^2 l}{A}+\frac{\cos^2 l}{B}\right)(G^2-L^2)+\frac{L^2}{2C}-nH \tag{1}$$

where A, B, and C are the principal momenta of inertia and n the mean motion of the primaries.

Two of the phase variables can be ignored and the torque-free motion is reduced to a conservative Hamiltonian system with only one degree of freedom. A qualitative study of the free rotation is made in Deprit.[14] However, since the unperturbed Hamiltonian [Eq. (1)] depends on l, it is impossible to apply the Lie-Deprit perturbation theory.[15]

For this reason, we use the angle-action variables $(\tilde{l},\tilde{g},\tilde{h},\tilde{L},\tilde{G},\tilde{H})$ given by Kinoshita[16] through the Hamilton-Jacobi equation for the Hamiltonian of Eq. (1). In that canonical variables set, the unperturbed Hamiltonian is independent of $\tilde{l}$, $\tilde{g}$, $\tilde{h}$, that is,

$$F_0 = F_0(\tilde{L},\tilde{G},\tilde{H})$$

The unperturbed problem solution is

$$\tilde{L}=\text{ct.},\qquad \tilde{G}=\text{ct.},\qquad \tilde{H}=\text{ct.}$$

$$\dot{\tilde{l}}=n_{\tilde{l}}^0,\qquad \dot{\tilde{g}}=n_{\tilde{g}}^0,\qquad \dot{\tilde{h}}=-n$$

where n is the mean motion of the primaries. The relations between the old and new variables and the expressions of $n_{\tilde{l}}^0$, $n_{\tilde{g}}^0$, can be found in Kinoshita's paper.[16]

Expansion of the Potential Function

The rotational motion of the satellite is, in general, coupled to its translational motion and one cannot expect a nonspherical satellite to remain in the plane of the revolution of even the primaries. However, when terms containing only second and third powers of the inverse distance remain in the differential equations, the equations of the translational motion can be separated from those of the rotational motion, thus forming an independent system. In this case, the equations of the rotational motion contain the coordinates of the mass center, which can be regarded as a known function of time.[17]

For this reason, we suppose the polar coordinates $r(t)$, $\theta(t)$ of the mass center (O_3) of the satellite S_3 as a known function of time and, thus, we can eliminate those terms not containing rotational variables from the Hamiltonian function.

The effect of the body O_i on the satellite, in McCullagh's form, is

$$V_i' = -k^2 m_i\left(\frac{m_3}{\zeta_i}+\frac{A+B+C-3I_i}{2\zeta_i^3}\right) \tag{2}$$

where A, B, and C are the principal momenta of inertia of S_3, I_i the momenta of inertia of S_3 respect to the axis joining O_3 with O_i, k^2 the gravitational constant, and $\zeta_i = |\overline{O_i O_3}|$.

Denoting by (λ_i,μ_i,ν_i) the direction cosines of $\overline{O_3 O_i}$ referred to the principal axes of inertia and eliminating the terms independent of the angular variables, the potential function has the form

$$V = -\frac{1}{2}k^2\sum_{i=1}^{2}\left\{\left(\frac{m_i}{\zeta_i^3}\right)[(A-B)(1-3\lambda_i^2) + (C-B)(1-3\nu_i^2)]\right\} \tag{3}$$

In order to express V as a function of Andoyer variables, we choose, as fixed plane O_3xy, the plane of motion of the centers of masses of the primaries; the axis O_3z is the perpendicular line to that plane, with the same sense of the angular momentum of the primaries motion. Finally, the axis O_3x coincides with the direction O_1O_2 (synodic system).

In those conditions, we have (see Fig. 1)

$$\begin{vmatrix}\lambda_i\\ \mu_i\\ \nu_i\end{vmatrix} = R_3^t(l)R_1^t(J)R_3^t(g)R_1^t(I)R_3^t(h-u_i)\begin{vmatrix}1\\0\\0\end{vmatrix} \tag{4}$$

where $\cos J = L/G$, $\cos I = H/G$, u_i is the angle between the direction O_3O_i and the positive rotating X axis, and $R_1(\xi)$, $R_3(\xi)$ are the matrices

$$R_1(\xi) = \begin{vmatrix}1 & 0 & 0\\ 0 & \cos\xi & -\sin\xi\\ 0 & \sin\xi & \cos\xi\end{vmatrix}$$

$$R_3(\xi) = \begin{vmatrix}\cos\xi & -\sin\xi & 0\\ \sin\xi & \cos\xi & 0\\ 0 & 0 & 1\end{vmatrix}$$

Inserting Eq. (4) into Eq. (3) and after simple but tedious calculations, we obtain

$$\tfrac{1}{2}[(A-B)(1-3\lambda_i^2)+(C-B)(1-3\nu_i^2)]$$

$$=\sum_{\rho\epsilon}\sum_{\gamma=0}^{2}\sum_{\beta=0}^{1}\sum_{\alpha=0}^{1} P_\beta Q_{\alpha\beta\gamma\rho\epsilon}\cos(2\alpha\delta_i+2\rho\beta h+\epsilon\gamma g) \tag{5}$$

where $\delta_i = h-u_i$, $P_0=(2C-A-B)/2$, $P_1=(A-B)$, and the indices ρ and ϵ take the values $+1$ and -1. The coefficients Q are given in the Appendix.

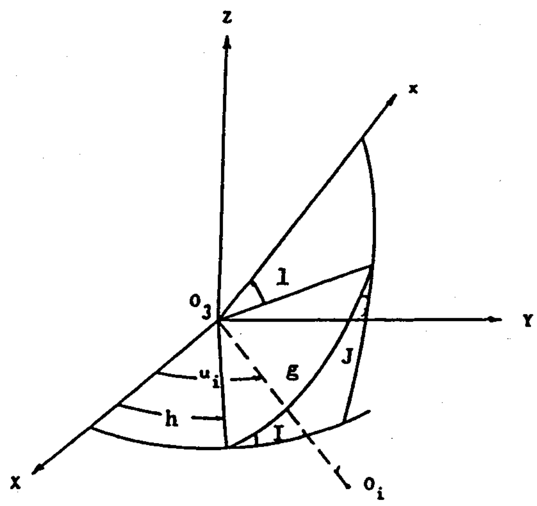

Fig. 1 Notation.

On the other hand, we have to express ζ_i, u_i, as functions of the polar coordinates $r(t)$, $\theta(t)$ of the center of masses O_3. We take the solution of the restricted three-body problem in the vicinity of L_4 given by Cid et al.[12] From that paper, we have

$$(\zeta_1)^{-k} = (r)^{-k}(1 - km\bar{A}) + o(m^2)$$

$$(\zeta_2)^{-k} = (r)^{-k}\left[\sum_{n\geq 0}\binom{-k/2}{n}\bar{C}^n\right] + o(m^2)$$

where $\bar{A} = (\cos\theta)/r$, $\bar{C} = (1/r - 2\cos\theta)/r$, and $m = m_2/(m_1 + m_2)$.

Taking into account the trigonometric relations between u_i, ζ_i and r, θ (see Fig. 2), we have

$$\cos(2\delta_1 + \Lambda) = \cos(2a_1 - \Lambda^*) = \cos\Lambda^*(1 - 2/rm\cos\theta)\cos 2\theta$$
$$+ U/rm\cos\theta + \sin\Lambda^*[(1 - 2/rm\cos\theta)\sin 2\theta + 2/rm\sin\theta]$$
$$\cos(2\delta_2 + \Lambda) = \cos(2a_2 + \Lambda^*) = (1 - \bar{C})[\cos\Lambda^*(\cos 2\theta$$
$$- 2/r\cos\theta + 1/r^2) + \sin\Lambda^*(\sin 2\theta - 2/r\sin\theta)]$$

where $\Lambda^* = 2h + \Lambda$.

Omitting terms containing $(1/r)^n$, $n \geq 1$, we obtain

$$\cos(2\alpha\delta_i + \Lambda) = C^i_{\alpha\beta\gamma\rho\epsilon}\cos(2\alpha h + \Lambda) + S^i_{\alpha\beta\gamma\rho\epsilon}\sin(2\alpha h + \Lambda) \quad (i = 1,2) \tag{6}$$

with

$$C^i_{0\beta\gamma\rho\epsilon} = Q_{0\beta\gamma\rho\epsilon}, \qquad S^i_{0\beta\gamma\rho\epsilon} = 0$$
$$C^i_{1\beta\gamma\rho\epsilon} = (\cos 2\theta)Q_{1\beta\gamma\rho\epsilon}$$
$$S^i_{1\beta\gamma\rho\epsilon} = (\sin 2\theta)Q_{1\beta\gamma\rho\epsilon}$$

Through the above relations, the potential is written in the following form:

$$V = -\sum \frac{m_i}{r^3}\sum_{\rho,\epsilon}\sum_{\gamma=0}^{2}\sum_{\beta=0}^{1}\sum_{\alpha=0}^{1} + P_\beta[C^i_{\alpha\beta\gamma\rho\epsilon}\cos(2\alpha h$$
$$+ 2\rho\beta l + \epsilon\gamma g) + S^i_{\alpha\beta\gamma\rho\epsilon}\sin(2\alpha h + 2\rho\beta l + \epsilon\gamma g)] \tag{7}$$

First-Order Perturbation

We assume that no commensurability among the mean motions of the angular variables $\tilde{l}$, $\tilde{g}$, $\tilde{h}$ exists. On the other hand, we have to express the disturbing function F_1 in terms of the angle-action variables. Nevertheless, we write F_1 as a function of Andoyer's variables. Besides, we will use the relations between those set of variables given in Kinoshita's paper.

By means of a canonical transformation of Lie-Deprit given by the generating function W, we eliminate the fast variables $\tilde{l}$, $\tilde{g}$, $\tilde{h}$. The new Hamiltonian is

$$F^*(\tilde{L}', \tilde{G}', \tilde{H}') = F_0^*(\tilde{L}', \tilde{G}', \tilde{H}')$$
$$+ \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} V d\tilde{l}' d\tilde{g}' d\tilde{h}' = F_0^* + F_1^*$$

The generating function W is the solution of the linear partial equation

$$\{F_0^*; W\} - \frac{\partial W}{\partial t} = F_1 - F_1^*$$

where $\{;\}$ stands for the Poisson bracket of F_0^* and W over the phase space.

It is easy to see that

$$\frac{1}{2\pi}\int_0^{2\pi} \begin{matrix}\cos\\ \sin\end{matrix}\, 2\alpha h d\tilde{h} = 0, \qquad \frac{1}{2\pi}\int_0^{2\pi} \begin{matrix}\cos\\ \sin\end{matrix}\, \gamma g d\tilde{g} = 0$$

and

$$S_{01011}\sin(2l) + S_{0101-1}\sin(2l)$$
$$+ S_{010-11}\sin(-2l) + S_{010-1-1}\sin(-2l) = 0$$

Therefore, the secular part of the Hamiltonian is

$$[V]_s = \frac{1}{2\pi}\int_0^{2\pi}[Q_0 + Q_1\cos(2l)]d\tilde{l}$$

with

$$Q_0 = -(2C - A - B)(3\cos^2 J - 1)(3\cos^2 I - 1)/(16r^3)$$
$$Q_1 = 3(A - B)\sin^2 J(3\cos^2 I - 1)/(16r^3)$$

Combining the expansion of the variables and the properties of the Jacobian elliptic functions, such as appear in Kinoshita's paper, we obtain

$$\frac{1}{2\pi}\int^{\pi}(3\cos^2 J - 1)d\tilde{l} = \frac{1}{2\pi}\int^{\pi}\left[3\frac{1 + eb}{(1 + e)b}dn^2\right.$$
$$\left.\times\frac{2K}{\pi}\left(l - \frac{\pi}{2}\right) - 1\right]d\tilde{l} = 3\frac{1 + eb}{(1 + e)b}\frac{E}{K} - 1 \tag{8a}$$

$$\frac{1}{2\pi}\int^{\pi}3(1 - \cos^2 J)\cos 2l d\tilde{l}$$
$$= \frac{3}{2\pi}\int^{\pi}\frac{b - 1}{b}\left[\frac{1}{1 - e}sn^2\frac{2K}{\pi}\left(l - \frac{\pi}{2}\right)\right.$$
$$\left.- \frac{1}{1 + e}cn^2\frac{2K}{\pi}\left(l - \frac{\pi}{2}\right)\right]d\ell = \frac{3}{eb}\left[1 - \frac{1 + eb}{1 + e}\frac{E}{K}\right] \tag{8b}$$

Therefore, we have

$$F_1^* = -\frac{1}{4r^3}(3\cos^2 I - 1)\left\{(2C - A - B)\left[3\frac{1 + eb}{(1 + e)b}\frac{E}{K} - 1\right]\right.$$
$$\left.- \frac{3}{4}(A - B)\frac{1}{eb}\left[1 - \frac{1 + eb}{1 + e}\frac{E}{K}\right]\right\} \tag{9}$$

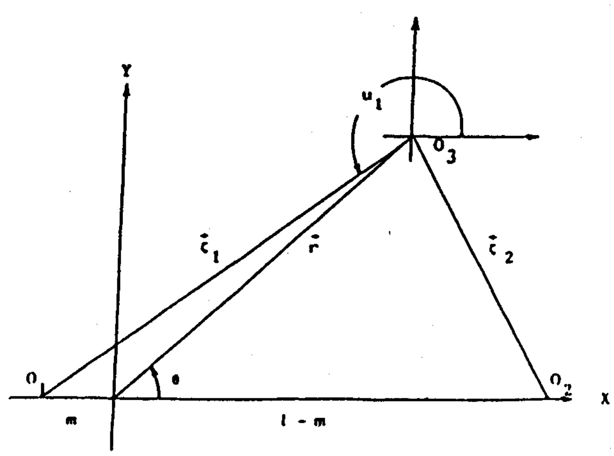

Fig. 2 Relationship between u_i, ζ_i, and θ.

where K and E are the complete elliptic integrals of the first and second class, respectively, $b=\tilde{G}^2/\tilde{L}^2$, and

$$e=\left(\frac{1}{A}-\frac{1}{B}\right)\Big/\left(\frac{1}{A}+\frac{1}{B}-\frac{1}{2C}\right)$$

It is worthy of notice that when the center of mass of S_3 is placed exactly at the libration point L_4, the above expression coincides with the averaged Hamiltonian given by Sidlichovsky.[7]

When the satellite is axisymmetric, $A=B$, $e=0$, and $E/K=1$. Therefore, it follows that

$$F_1^*=-(C-A)(3\cos^2 I-1)(3\cos^2 J-1)/(4r^3)$$

This Hamiltonian has been studied in Elipe and Ferrer.[11]

From the properties of the functions e, b and the complete elliptic integrals E, K, Eqs. (8a) and (8b) can be developed into a power series in e^2; up to the second power of e, we have

$$\left[3\frac{1+eb}{(1+e)b}\frac{E}{K}-1\right]=\left(\frac{3}{\tilde{b}^2}-1\right)-\frac{3}{8\tilde{b}}(3+\tilde{b})e^2 \qquad (10a)$$

$$\frac{3(A-B)}{4eb}\left[1-\frac{1+eb}{(1-e)}\frac{E}{K}\right]$$
$$=-\frac{3(2AB-AB-BC)}{16C}\frac{1}{\tilde{b}}(3+\tilde{b})e^2 \qquad (10b)$$

with $\tilde{b}=\tilde{G}^2/\tilde{L}^2$.

Then, the expression F_1^* yields

$$F_1^*=-\frac{1}{4r^3}(3\cos^2 I-1)\left[\frac{2C-A-B}{4}(3\cos^2\tilde{J}-1)\right.$$
$$\left.-\frac{3}{32C}(2C^2-3AC-3BC+4AB)\sin^2\tilde{J}(3+\tilde{b})e^2\right] \qquad (11)$$

The mean motions $n_{\tilde{l}}$, $n_{\tilde{g}}$, $n_{\tilde{h}}$, of $\tilde{l}$, $\tilde{g}$, $\tilde{h}$, are, up to the second order in e,

$$n_{\tilde{l}}=n_{\tilde{l}}^0+\Delta n_{\tilde{l}},\quad n_{\tilde{g}}=n_{\tilde{g}}^0+\Delta n_{\tilde{g}},\quad n_{\tilde{h}}=-n+\Delta n_{\tilde{h}}$$

where

$$\Delta n_{\tilde{l}}=-\frac{1}{2r^3}(3\cos^2 I-1)\frac{\tilde{L}}{\tilde{G}^2}\left[\frac{3}{4}(2C-A-B)\right.$$
$$\left.+\frac{3}{32C}(2C^2-3AC-3BC-4AB)(3+\tilde{b})e^2\right]$$

$$\Delta n_{\tilde{g}}=-\Delta n_{\tilde{h}}\cos I+\frac{1}{2r^3}(3\cos^2 I-1)\frac{\tilde{L}^2}{\tilde{G}^3}$$

$$\times\left[\frac{3}{4}(2C-A-B)+\frac{3}{32C}(2C-3AC-3BC-4AB)(3+\tilde{b})c^2\right]$$

$$\Delta n_{\tilde{h}}=-\frac{3}{2r^3}\frac{\tilde{H}}{\tilde{G}^2}\left[\frac{2C-A-B}{4}(3\cos^2\tilde{J}-1)\right.$$
$$\left.-\frac{3}{32C}(2C^2-3AC-3BC-4AB)\sin^2\tilde{J}(3+\tilde{b})c^2\right]$$

The mean motion of rotation along the axis O_3z is

$$\Omega=\Omega^0+\Delta\Omega=\frac{d\tilde{l}}{dt}+\frac{d\tilde{g}}{dt}\cos\tilde{J}+\frac{d\tilde{h}}{dt}\cos I\cos\tilde{J}$$
$$=\Omega^0+\Delta n_{\tilde{l}}+\cos\tilde{J}(\Delta n_{\tilde{g}}+\Delta n_{\tilde{h}}\cos I)=\Omega^0-\frac{\tilde{L}}{\tilde{G}^2}\sin^2\tilde{J}$$
$$\times\left\{\frac{1}{2r^3}(3\cos^2 I-1)\left[\frac{3}{4}(2C-A-B)\right.\right.$$
$$\left.\left.+\frac{3}{32C}(2C^2-3AC-3BC+4AB)\cdot(3+\tilde{b}^2)e^2\right]\right\}$$

When the deviation from the axial symmetry is small and the inclinations I, $\tilde{J}$ are also small, the oblate ($C>A,B$) or prolate ($C<A,B$) shape of the satellite has influence on the rotational motion. In the oblate case, $n_{\tilde{h}}<0$ and, thus, the variable h has a retrograde variation; whereas in the prolate case, $n_{\tilde{h}}>0$ and the variable h has a reduction in its retrograde mean motion, referred to the synodic system.

When the satellite S_3 is prolate (oblate), the precessional mean motion of the plane normal to the angular momentum vector of S_3 is smaller (greater) than that of the undisturbed torque-free motion.

Appendix

$Q_{00011}=Q_{0001-1}=Q_{000-11}=Q_{000-1-1}$
$=(1/16)(3\cos^2 J-1)(3\cos^2 I-1)$

$Q_{00011}=Q_{0001-1}=Q_{000-11}=Q_{000-1-1}=-(3/16)\sin 2J\sin 2I$

$Q_{00011}=Q_{0001-1}=Q_{000-11}=Q_{000-1-1}=(3/16)\sin^2 I\sin^2 J$

$Q_{10011}=Q_{1001-1}=Q_{100-11}=Q_{100-1-1}$
$=(3/16)\sin^2 I(3\cos^2 J-1)$

$Q_{10111}=Q_{101-11}=(3/8)\sin 2J\sin I(\cos I+1)$

$Q_{1011-1}=Q_{101-1-1}=(3/8)\sin 2J\sin I(\cos I-1)$

$Q_{10211}=Q_{102-11}=(3/16)\sin^2 J(1+\cos I)^2$

$Q_{1021-1}=Q_{102-1-1}=(3/16)\sin^2 J(1-\cos I)^2$

$Q_{01011}=Q_{0101-1}=Q_{0101-1}=Q_{010-1-1}$
$=-(3/32)\sin^2 J(3\cos^2 I-1)$

$Q_{01111}=Q_{011-1-1}=-(3/16)\sin 2I\sin J(\cos J+1)$

$Q_{0111-1}=Q_{011-11}=-(3/16)\sin 2I\sin J(\cos J-1)$

$Q_{01211}=Q_{012-1-1}=-(3/32)\sin^2 I(1+\cos J)^2$

$Q_{0121-1}=Q_{012-11}=-(3/32)\sin^2 I(1-\cos J)^2$

$Q_{11011}=Q_{1101-1}=Q_{110-11}=Q_{110-1-1}$
$=-(9/32)\sin^2 J\sin^2 I$

$Q_{11111}=(3/8)\sin J\sin I(\cos J+1)(\cos I+1)$

$Q_{1111-1}=(3/8)\sin I\sin J(\cos I-1)(\cos J-1)$

$Q_{111-11}=(3/8)\sin I\sin J(\cos I+1)(\cos J-1)$

$$Q_{111-1-1} = (3/8)\sin I \sin J(\cos I - 1)(\cos J + 1)$$

$$Q_{11211} = -(3/32)(1+\cos I)^2(1+\cos J)^2$$

$$Q_{1121-1} = -(3/32)(1-\cos I)^2(1-\cos J)^2$$

$$Q_{112-11} = -(3/32)(1+\cos I)^2(1-\cos J)^2$$

$$Q_{112-1-1} = -(3/32)(1-\cos I)^2(1+\cos J)^2$$

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